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Space-charge effects on nonlinear amplification of inverse bremsstrahlung electron acceleration

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The effect of space charge on the nonlinear amplification of inverse bremsstrahlung electron acceleration is investigated. It is shown that for a highly relativistic electron beam the space-charge effect modifies only slightly the energy gain.

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The possibility of using intense lasers to accelerate electrons to extremely high energies has been attracting the attention of the scientific community for the last two decades. Different schemes have been proposed to tap the energy from the laser field, which is not available in the isolated electron-photon interaction. One of the possibilities is to use a dielectric medium, such as a plasma, to excite an electrostatic wave which can then directly accelerate the electrons [1]. However, it is in principle more convenient to avoid the intervention of such a medium and indeed many plasma-free alternatives have also been considered [2–8]. Kawata and co-workers have proposed a scheme in which a magnetostatic [4] or electrostatic field [5] is employed perpendicularly to the directions of the laser and particle beams. Net energy gain can be obtained in one cycle of the electromagnetic field. A quite relevant modification of this scheme was proposed by Hussein and Pato [6]. By properly alternating the sign of the electrostatic field at optimally determined positions, the energy of the electrons can be made to increase continuously. For a relativistic beam and for parameters of interest, the distance between the positions of sign inversion is large enough (tens of centimeters) to make the electrostatic structure technically feasible. This scheme has been dubbed NAIBEA for nonlinear inverse bremsstrahlung electron acceleration. A Hamiltonian formalism has been developed to allow the study of more general cases, including the simultaneous presence of electrostatic and magnetostatic fields and a finite laser pulse [9].

The calculation of the energy gain is usually based upon a single-particle model, although Kawata and coworkers have also carried out particle-in-cell simulations [4,5]. Actually, since the NAIBEA scheme is meant to be a high energy booster, acting on an already highly relativistic electron beam, one would expect that space-charge effects play a negligible role in the parallel energy gain. However, it turns out that the electrostatic field slightly deforms the beam trajectory and the small but finite perpendicular drift plays a crucial role in maintaining the proper phase of the laser field, with respect to the particle, for net energy gain. A somewhat similar problem occurs in the free-electron laser [10,11]. We have therefore decided to investigate the effect of the space-charge field on the NAIBEA mechanism. We do not consider the related problem of beam dispersion, which is similar to the one for conventional particle accelerators.

To qualitatively discuss the effect of the space-charge potential, it is convenient to refer to the Hamiltonian formalism developed in Ref. [9]. Using the phase of the electromagnetic wave, $\varphi = ct - kz$, as an integration variable, the equation for the rate of change of the relativistic factor γ is [Eq. (15) of [9]]

$$\frac{d\gamma}{d\varphi} = \frac{1}{u}\frac{dQ}{d\varphi}(E_{\text{laser}} + E_{\text{app}}), \tag{1}$$

where $d\mathbf{Q}/d\varphi = -\mathbf{A}$. $\mathbf{E}_{\text{laser}} = -\partial\mathbf{A}/\partial t$ is the laser electric field, and $\mathbf{E}_{\text{app}}$ is the externally applied electrostatic field (see Fig. 1). The variable u is defined as $u = \gamma - p_x/mc$, where p_x is the longitudinal component of the canonical momentum. Taking into account the space-charge potential $\phi(x)$ in the particle Hamiltonian and following the derivation outline in Ref. [9], we obtain

$$u^2 = u_0^2 + 2E_{\text{app}}Q + 2\int_0^{\varphi} d\varphi' u(\varphi')\frac{d\phi}{dx}. \tag{2}$$

In Eqs. (1) and (2) energy is normalized to mc^2, distances to $1/k$ and time to $1/\omega$, where k and ω are respectively

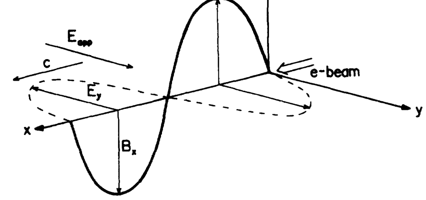

FIG. 1. Configurations of the laser-electron-beam system in the presence of an externally applied electrostatic field.

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the wave number and wave frequency, and the vector potential to mc/e. Clearly, for substantial net energy gain it is necessary that the left-hand side of Eq. (1) remains always positive and that u be made as small as possible. Actually $u \to 0$ as the particle speed approaches c. However, we see from Eq. (2) that the value of u depends directly on the self-consistent space potential $\phi(x)$. The externally applied electrostatic field $\mathbf{E}_{\text{app}}$ is used to fine tune the sign of p_y with respect to $\mathbf{E}_{\text{laser}}$ and its value is many orders of magnitude smaller than the value of the latter. Therefore even a weak space-charge field can have a significant effect on the value of u.

To take into account the space-charge field self-consistently, we use the phase-average bunching approximation usually employed in free-electron lasers to model the same effect [10,11]. The equation of motion for each electron is

$$\frac{d\mathbf{p}}{dt} = -e(\mathbf{E} + \mathbf{v} \times \mathbf{B}), \tag{3}$$

where the electric field is given by

$$\mathbf{E} = E_{\text{app}}\hat{\mathbf{a}}_y + E_{\text{laser}}\hat{\mathbf{a}}_y - \frac{d\phi}{dx}\hat{\mathbf{a}}_x, \tag{4}$$

$$E_{\text{laser}} = -E_{y0}\sin\varphi, \quad \varphi = k(ct - x). \tag{5}$$

Integrating the equations of motion and using Eqs. (4) and (5), we obtain

$$\frac{d\varepsilon}{dt} = ev_x\frac{d\phi}{dx} - ev_yE_y, \tag{6}$$

where the particle energy is given by

$$\varepsilon(t) = m\gamma c^2 + eE_{\text{app}}y(t), \tag{7}$$

$$p_xc = m\gamma c^2 + eE_{\text{app}}y + K_1 + \int (c - v_x)e\frac{d\phi}{dx}dt, \tag{8}$$

and

$$p_yc = -\frac{e}{k}E_{y0}\cos\varphi - eE_{\text{app}}t + K_2, \tag{9}$$

where K_1 and K_2 are constants.

For a relativistic electron beam, $\beta_x \approx 1$, the last term in Eq. (8) can be neglected and the expression for p_xc is the same as without space charge [6]. This is consistent with the electric field of a relativistic charged particle being mainly perpendicular to the direction of its velocity. However, the effect of the space-charge potential in the equations for the energy evolution cannot be neglected. Following Refs. [10,11], we model the space-charge effect assuming that the particle beam consists of N charge sheets per wavelength of the laser field, with an initially uniform distribution such that their initial positions are given by $x_j(t=0) = (2\pi/k)[(j-1)/(N-1) - 1/2]$. As time evolves, each sheet will change its position and therefore its phase relative to the electromagnetic wave. The "fine tuning" achieved with the externally applied electrostatic field is modified by the field produced by the sheets. Considering only the dominant longitudinal displacement, we can represent the positions of the charge sheets in terms of their phases, i.e., the beam charge density is written as

$$n(\varphi) = \frac{2\pi\bar{n}_e}{N}\sum_{j=1}^{N}\delta(\varphi - \varphi_j), \tag{10}$$

where $\bar{n}_e$ is the average beam density and the particle phases are taken in the interval $-\pi \le \varphi_j \le \pi$. Expanding $n(\varphi)$ in a Fourier series and using the relation $\partial_\varphi/\partial x = k$, we can integrate once Poisson's equation, $\nabla^2\phi = -nq/\varepsilon_0$, to obtain

$$\frac{d\phi}{dx} = i\frac{\bar{n}_e e}{\varepsilon_0 k}\sum_{m=1}^{\infty}\frac{\langle e^{-im\varphi_j}\rangle}{m}e^{im\varphi} + \text{c.c.}, \tag{11}$$

where

$$\langle e^{im\varphi_j}\rangle = \frac{1}{N}\sum_{j=1}^{N}e^{-i\varphi_j} \tag{12}$$

is the so-called *bunching parameter* [10,11].

Substituting Eq. (11) into Eq. (6) and normalizing distances to a characteristic length l, time to l/c, energy to mc^2, and electric field to mc^2/el, we obtain from Eqs. (6)–(9) the evolution equations for particle i in normalized form

$$\frac{dx_i}{dt} = \frac{p_{x_i}}{\gamma_i}, \tag{13}$$

$$\frac{dy_i}{dt} = \frac{p_{y_i}}{\gamma_i}, \tag{14}$$

and

$$\frac{d\varepsilon_i}{dt} = E_{y0}\frac{p_{y_i}}{\gamma_i}\sin\varphi_i - 2\delta\,\text{Im}\left\{\sum_{m=1}^{\infty}\frac{\langle e^{-im\varphi_j}\rangle}{m}e^{im\varphi}\right\}, \tag{15}$$

coupled with the equations

$$p_{x_i} \simeq \gamma_i + E_{\text{app}}y_i + K_{1i}, \tag{16}$$

$$p_{y_i} = -\frac{E_{y0}}{k}\cos\varphi_j - eE_{\text{app}}t + K_{2i}, \tag{17}$$

$$\gamma_i = \varepsilon_i - E_{\text{app}}y_i, \tag{18}$$

and

$$\varphi_i = k(t - x_i). \tag{19}$$

The effect of the space charge appears through the parameter

$$\delta = \frac{\omega_b^2}{\omega^2}\left(\frac{\omega}{c/l}\right) \tag{20}$$

in Eq. (15), where $\omega_b^2 = \bar{n}_e e^2/m\varepsilon_0$ is the beam plasma

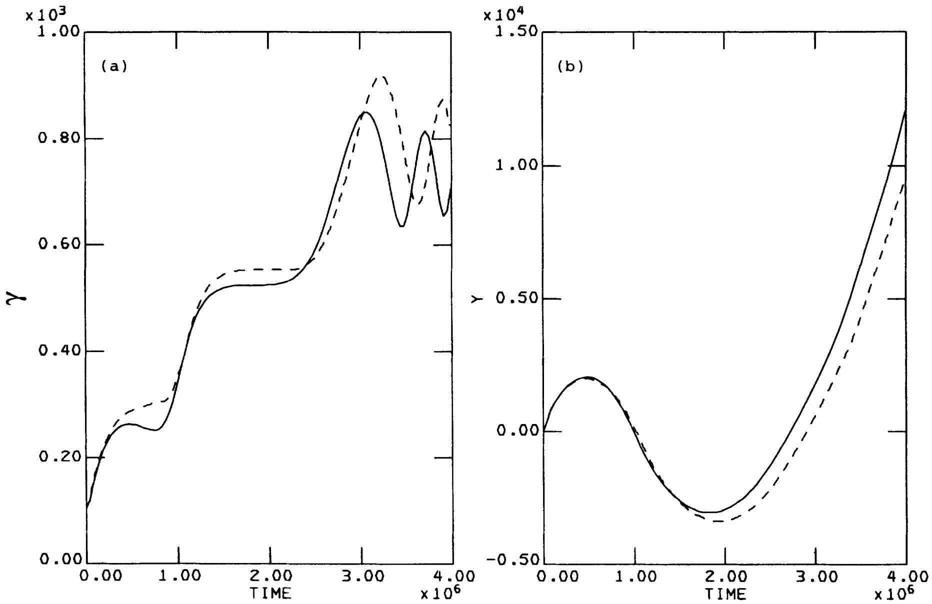

FIG. 2. Temporal profiles of $\gamma(t)$ (a) and perpendicular displacement $y(t)$ (b) of the central "tag" particles in a simulation of NAIBEA with $N = 19$ particles/wavelength. The full lines correspond to $\delta = 10^{-8}$ and the dashed lines to $\delta = 2.0 \times 10^{-4}$. Time is normalized to $\pi/16kc$.

frequency.

Equations (13)–(15) are numerically integrated for given initial conditions; the N particle sheets are uniformly distributed in one wavelength and the central particle $[x(0) = 0]$ is chosen as a "tag" particle. The sign of the externally applied electrostatic field $\mathbf{E}_{\rm app}$ is changed at the appropriate phase of the particle. At each change of the sign of $\mathbf{E}_{\rm app}$, all constants K_{1i} and K_{2i} have to be altered to guarantee continuity of momentum. We find that the results are independent of the value of N for $N \gtrsim 15$.

We now consider the specific example discussed by Hussein and Pato [6], i.e., a $\lambda = 10$ μm laser with intensity $I = 3.5 \times 10^{15}$ W/cm^2 and an electrostatic field whose amplitude is 4.28×10^{-5} times the amplitude of the laser electric field. The initial electron velocity is $v_{0x} = 0.9999c$ and the beam is injected at an angle of 0.608° with respect to the direction of propagation of the electromagnetic wave. Therefore the initial value of the relativistic mass factor is $\gamma_0 = 106.82$. We choose the normalizing scale length $l = \lambda/32$ and $N = 19$ particles/wavelength.

In Fig. 2 we show the results for $\delta = 10^{-8}$ (full lines) and $\delta = 2 \times 10^{-4}$ (dashed lines), with one inversion in the sign of $\mathbf{E}_{\rm app}$. For a tenuous beam ($\delta = 10^{-8}$), the results agree well with the single-particle calculations carried out by Hussein and Pato [6]. For a high-density beam ($\delta = 2 \times 10^{-4}$), with beam density of the order of that considered by Kawata and co-workers [4], there is a somewhat surprising small increase in the energy gain

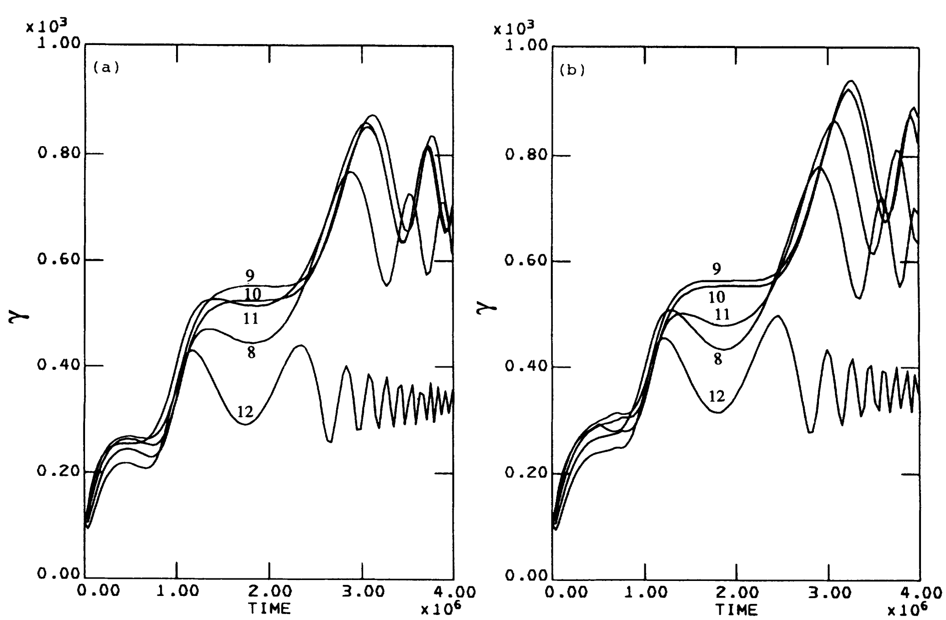

FIG. 3. Temporal profiles of $\gamma(t)$ for the central "tag" particle, 10, and the next two trailing it, 8 and 9, and leading it, 11 and 12, for $\delta = 10^{-8}$ (a) and $\delta = 2.0 \times 10^{-4}$ (b). Time is normalized to $\pi/16kc$.

of the "tag" particle. This is due to a slight decrease in the perpendicular excursion of the particle, as shown in Fig. 2(b), caused by the self-consistent interaction. In Fig. 3 we show the time evolution of γ for five particles, the "tag" and the next two leading and next two trailing it. The sign of $\mathbf{E}_{\rm app}$ is changed at the optimal phase ($\varphi = 3\pi/2$) of the "tag" particle with respect to the laser field. We see that only the foremost particle is not affected by the sign change, both in the low-density [Fig. 3(a)] and high-density [Fig. 3(b)] cases. Therefore the space-charge effects do not spoil the overall energy gain.

Concluding, we have shown that NAIBEA works well also for conditions such that space-charge effects are relevant. The calculation has been carried out under the phase-average bunching and highly relativistic beam approximations. However, the results are supported by particle-in-cell simulations which will be reported soon. For the practical implementation of this scheme, the rather difficult problem of how to keep the laser focused for the whole extension of the laser-beam interaction region has still to be solved. We envisage a special optical system with a large number of laser beams propagating parallel to the electron beam. Each laser beam is diverted at a proper location and focused onto the electron beam. Therefore a more realistic simulation will have to be carried out considering the nonlinear acceleration process in an electromagnetic wave with a slowly varying amplitude [2].

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